

COSPAR - IAU - LSI COLLOQUIUM ON
LUNAR DYNAMICS
AND OBSERVATIONAL COORDINATE SYSTEMS

REVISED ABSTRACTS

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EDITED BY
MICHAEL MOUTSOULAS

LSI CONTRIBUTION
#135

LUNAR SCIENCE INSTITUTE
HOUSTON, TEXAS

JANUARY 1973

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Preface

The Colloquium on "Lunar Dynamics and Observational Coordinate Systems" held at the Lunar Science Institute* in Houston from January 15-17, 1973, gave the participants the opportunity to exchange information on recent developments in the field, which have indeed been considerable. Moreover, the necessity for coordination of efforts and better communication among investigators who work with different methods and techniques was amply emphasized. The two panel discussions, on the establishment of a fundamental reference system and on the recently developed lunar ephemerides clearly indicated the beneficial role of such coordination.

While the full text of the papers presented to the Colloquium will appear in a special issue of "The Moon" by the end of this year, we felt that it would be advantageous to provide our colleagues with a brief volume of revised abstracts on the topics discussed, as early as possible. Since the Commission 17 of the IAU has planned a discussion on the selenographic

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control points during the General Assembly in Sydney next August, appearance of this informative summary before that time may be helpful.

It is a pleasant duty to express, on behalf of the participants, our deep appreciation and thanks to Dr. J. Derral Mulholland who so ably planned and chaired the Colloquium, and to Dr. J. W. Chamberlain and the Lunar Science Institute staff for coordinating and hosting the meeting in such a splendid way.

Houston, January 1973

Michael Moutsoulas

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CONTRIBUTION TO AN ANALYTICAL LUNAR THEORY

A. Bec, J. Kovalevsky and Cl. Meyer

During the last year, the project of an analytical lunar theory was revived. All programs for computations on literal trigonometric series with rational coefficients were rewritten for an IBM 360-65 with new principles as far as the internal structure of series is concerned. The computational loops have been programmed and results of the calculations for the first loops will be presented. The method is essentially based on the theory derived by Chapront and L. Mangeney.

To prepare the successive approximations, all the expressions are computed by incremental formulae. A special device has been prepared in order to keep the denominators only in those terms where their development in power series affects the convergence of the coefficients.

SELENOCENTRIC COORDINATES OF THE APOLLO RETROREFLECTORS

P. L. Bender, D. G. Currie, J. E. Faller, W. M. Kaula,
J. D. Mulholland, E. C. Silverberg, and J. G. Williams

The selenocentric coordinate system with respect to which the Apollo 11, 14, and 15 laser retroreflector locations are being determined is described. It is based on the mean direction of the earth and the mean lunar spin axis. Current values of the retroreflector coordinates are given, and their reliability is discussed. The libration theory which was used includes the effects of third degree terms in the lunar mass distribution, as calculated recently by Eckhardt. Potential applications of this work to lunar cartography are discussed.

PRECISE DETERMINATION OF LUNAR BASELINES AND LIBRATION
BY DIFFERENTIAL VLBI OBSERVATIONS OF ALSEPS

C. C. Counselman III, H. F. Hinteregger,
R. W. King, and I. I. Shapiro

Observations of the signals from the ALSEP transmitters in a differential interferometric mode [see Science, vol. 178, pp. 607-608 (10 November 1972)] are being used to determine the relative positions of the ALSEPs on the surface of the moon and the motion of the moon about its center of mass. Such observations were begun by us with the Spacecraft Tracking and Data Network (STDN) operated by the NASA Goddard Space Flight Center on a trial basis in early 1971. Determinations of ALSEP positions from the early observations suffered from instrumental errors amounting to some tens of meters on the lunar surface. We have since developed and used successfully a new type of differential VLBI receiver which reduces both random and systematic instrumental errors to negligible levels. With less than 1 second of averaging time the random measurement "noise" is the equivalent of 4 cm on the lunar surface. Instrumental systematic errors are even smaller than this. Errors in inferred ALSEP positions due to uncorrected differential atmospheric and ionospheric effects are also negligible, and the main limitation on the accuracy of the results is not set by uncertainty in the present

lunar orbital ephemeris. A 300-meter error in the moon's orbital longitude causes a 1-meter error in the relative positions of ALSEPs inferred from one day's observation. Even assuming that no improvement can be made in our knowledge of the moon's orbit, we expect to be able to determine the relative latitudes and longitudes of the ALSEPs to within 1 meter, and the orientation of the moon within 1 second of selenocentric arc, in a time short compared to the libration period. Our present knowledge of the lunar orbit may be improved significantly by making differential measurements between ALSEPs and quasars using a wideband VLBI system which we have also developed.

NUMERICAL SERIES FOR THE VARIATIONS
OF THE MOON'S COORDINATES

André Deprit

The main problem of lunar theory has been solved analytically to an accuracy comparable to the uncertainty affecting laser observations.

The literal series have been converted into multivariate Fourier series; the coefficients for the longitude and latitude on the one hand and for the sine parallax on the other hand have been computed so far as to make the truncation errors less than 10^{-4} and 10^{-6} seconds of arc respectively. For the first time in the history of lunar theory, corresponding series are produced for the variations of the coordinates with respect to the constants of the theory. This makes possible continuous updating of the Analytical Lunar Ephemeris by comparison with observations.

OBSERVING TOTAL SOLAR ECLIPSES FROM NEAR
THE EDGE OF THE PREDICTED PATH

David W. Dunham and Joan Bixby Dunham

Making a best fit of the contacts recorded during a well-observed grazing occultation of a star with the profile predicted from C. B. Watts' Marginal Zone of the Moon shows that the location of Watts' datum with respect to the star can be determined to an accuracy of about $\pm 0''.02$. We have investigated the possibilities of making visual and photographic observations of total eclipses of the Sun from near the edge of the predicted path to try to attain similar accuracies for the relative diameters and celestial latitudes of the Sun and Moon. With the improvements in our knowledge of the constants of the Moon's motion which are being made with the help of photoelectric occultation timings, grazing occultations, and laser rangings, it should be possible to use Solar eclipse observations to improve the values of the orbital elements of the Earth's orbit and perhaps certain astrometric constants. Elaborate expeditions have obtained very accurate relative positional information during previous Solar eclipses, but these all observed from close to the central line, or from outside the path of totality.

As seen from a few miles inside the predicted edge of the path of a total Solar eclipse, totality will last for an

appreciable fraction (typically 20% to 40%) of the central line duration, while the durations of limb phenomena are magnified by an order of magnitude over their central line values. Although we are primarily interested in observations of Bailey's beads and timings of second and third contacts for positional information, other limb phenomena, including the flash spectrum and shadow bands, are greatly enhanced.

During the Solar eclipse of 10 July 1972, visual timings of second and third contact were made at several locations just inside the northern and southern edges of the path of totality in eastern Canada, and some useful photographic data were obtained in spite of considerable cloudiness. Observed durations of totality ranged from about 15 seconds to a minute. The chromosphere remained visible throughout totality. Preliminary analysis shows that the shadow passed about 3 kilometers north-east of its predicted path, implying a shift of about $0''.6$ in the relative celestial latitudes of the Sun and Moon. A thorough analysis of these observations is in progress at the U. S. Naval Observatory.

CASSINI'S LAWS

Donald H. Eckhardt

Two hundred and eighty years ago, Jean Dominique Cassini disclosed his finding that the rotational motion of the moon could be neatly described by the superposition of two uniform motions, a prograde rotation of the moon about its polar axis and a retrograde precession of the moon's equator along the ecliptic. The description of these motions is now called Cassini's laws. As usually stated in the recondite prose of astronomy, these laws seem somewhat mysterious; and their standard dynamical explanation is just complicated enough mathematically that the mystery is not completely dissipated. But Cassini's laws are quite easy to comprehend for they are simple to observe and reconstruct, and their dynamical explanation can be done in words and pictures so that the physical principles are not lost in a maze of mathematics.

THE OCCULTATION PROGRAM OF MCDONALD OBSERVATORY

David S. Evans

Since 1968 an assiduous program of photoelectric observation of occultations of stars by the Moon has been pursued at McDonald Observatory. A total of about 600 events has been observed of which 254 have been published and a second list is in preparation. Timings derived from the reductions have errors of the order of 1 millisecond corresponding to a positional uncertainty in the lunar limb of typically 80 centimeters. Results are routinely communicated to the ephemerides offices.

In about 140 events the fringe pattern due to diffraction by the lunar limb is sufficiently well defined to permit a determination of the slope near the point of occultation. A statistical discussion of these data is given. In all except a few cases, the slopes are numerically less than 15° , though 7 cases with slopes between 30° and 40° have been found. The relations between numbers of observations and contact angle, and between errors of slope determination and contact angle are discussed. The distribution of slope data with contact angle seems adequately explained. When slope data are collected by position angle consistently large and consistently small values show a tendency to group in a pattern suggesting a connection with observed large scale features on the lunar limb.

The influence of lunar limb irregularities of a scale of a few meters on observed diffraction patterns and inferred timings is discussed. Multichannel observations should be of value in removing ambiguities. The use of occultation observations for the discovery of multiple stars and for the measurement of angular diameters of stars is mentioned. Future developments proposed for the project are considered.

The project has involved contributions by a considerable number of individuals from the staff and student body at Austin, Texas, from the staff at McDonald Observatory, from visiting scientists and from the Laser Ranging Group. These contributions are acknowledged in the paper. The work has been supported by NSF Grants GP-21204 and GP-32263X.

PRELIMINARY LIST OF FUNDAMENTAL CRATERS
FOR ESTABLISHING A LUNAR COORDINATE SYSTEM*

M. Froeschle and Th. Weimer

The presented list is the result of a decision of the Working Group 2 "Figure, Rotation and Observed Positions of the Moon" of I.A.U.'s Commission 17, decision taken at Newcastle-upon-Tyne in March 1971.

All the existing catalogues of craters have two principal inconveniences:

a) They have only a small number of common craters, therefore it is very difficult, if not impossible, to compare them one to the other.

b) The positions of the craters are not fundamental: in a manner or another they are related to the position of Mösting A (which is fundamental) and not to the inertial axis of the Moon.

As early as 1960, Th. WEIMER presented at the I.A.U. Symposium 14 in Leningrad (p. 79-84) a list of 135 craters intended to be measured on plates with a star field (Plates taken with Markowitz' camera, for instance).

*For a copy of the list apply to the authors,

Observatoire de Paris
61, Avenue de l'Observatoire
75014 Paris (France)

The proposal came too soon and, excepting Schrutka-Rechtenstamm (Ann. Univ. Sternw. Wien 1966, Bd. 26, Heft 6) nobody included these craters in his catalogue.

The new list was elaborated after many consultations with Dr. GAVRILOV and Dr. MOUTSOULAS. It differs from the previous list: only 38 craters are common, for the list of 1960 had a great amount of big craters, which could be easily measured on plates obtained with 4m. focus refractors.

The list comprises two parts:

1) A list A of 60 craters of a diameter superior to 8 km. These craters can be measured on Moon pictures up to 5 cm.

2) A list B of 140 craters of diameters from 3 to 8 km.

The craters of both lists are spread as uniformly as possible over the visible disk. An observer, whatever may be the focal length of his refractor, will be able to measure a sufficient number of craters that are common with other observers. Thus, future fundamental catalogues can be easily compared. Craters are fundamental, if their coordinates are referred to a lunar inertial system, i.e., they must be connected to stars.

The craters come from various lists and catalogues. They were chosen on account of their position (uniform distribution) and of their photographic qualities under different illuminations. 10 plates with 3, 5 cm. Moon and 10 with 18 cm. Moon

were examined. They were notated for the possibility of easily identifying them, for their shape and eventually the phase effect, for their more or less good definition. The mean of the 10 plates was taken, and only the best craters were selected (about 1 of 4).

The proposed list is not definitive; it should be completed by craters chosen and intended to be measured on spacecraft photographs. This amended list should be submitted to the General Assembly of I.A.U. in Sydney.

A Unified Selenodetic System, i.e., a catalogue giving the positions of fundamental craters, can be developed later on from the comparison of the different catalogues.

SELENODETIC CONTROL DERIVED FROM APOLLO
METRIC PHOTOGRAPHY

Raymond Helmering

The data reduction of the metric photography from the Apollo missions is progressing in an orderly fashion within the Defense Mapping Agency (DMA). The data from all three Apollo missions is ultimately to be utilized for development of a lunar control network covering approximately 20% of the lunar surface. In this paper, the status of the data reduction from the Apollo 15 mission is summarized. More specifically, the evaluation of system parameters, proposed control generation plan, and the anticipated characteristics of the network are discussed.

COMPOSITE LUNAR GRAVITY FIELDS*

C. A. Lundquist, R. H. Gay and G. E. O. Giacaglia

Three desiderata were established for a composite lunar gravity field. First, the composite should preserve the values of some adopted set of low degree spherical harmonic coefficients obtained from analysis of long orbital arcs. Second, the composite field should incorporate short wavelength detail from one of the available near-side gravity maps. Third, the composite should preserve the far-side gravity profile given by the adopted set of low degree spherical harmonic coefficients. The generation of spherical harmonic expansions to satisfy these desiderata was accomplished through the spherical sampling function formalism for degree $N = 36$, involving $(N + 1)^2$ or 1369 sample points distributed over a spherical reference surface (Giacaglia and Lundquist, 1972). For that fraction of the moon covered by a near-side map, the radial gravity anomaly values at the sample points were tabulated from the map. For the remainder of the moon, radial gravity anomaly values at the sample points were calculated from the adopted set of low degree spherical harmonic coefficients.

*This research was supported in part by grant NRL/ONR N00014-71-A-0110-0004.

The low degree harmonic coefficients corresponding to this composite set of sample point values were next derived using a transformation matrix prescribed by the sampling function formalism (Gay, 1973). These harmonic coefficients were compared with the adopted low degree coefficients, and the gravity values for the points covered by the near-side map were adjusted to preserve the adopted coefficients. These adjusted values were subsequently combined with the previously calculated gravity values for the remainder of the moon, and the same transformation applied again to derive coefficients for a spherical harmonic representation truncated at some degree between 14 and 20. The resulting composite gravity representation satisfies the desiderata to an acceptable approximation.

This general procedure was applied for several choices of low degree harmonic coefficients, (e.g. Liu and Laing, 1971, truncated at 8, 8 and at 4, 4; Michael and Blackshear, 1972, truncated at 8, 8; Ferrari, 1972, truncated at 4, 4). Two near-side gravity maps were used (one from JPL corresponding to Sjogren, et al., 1971; a second from Wong, et al., 1971). Spherical harmonic coefficients for composite gravity fields were generated for several combinations of these and other choices.

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RESULTS FROM LUNAR OCCULTATIONS

L. V. Morrison

About 40,000 visual and photo-electric timings of lunar occultations for 1943-1972 have been analyzed to find corrections to the following:

1. Brown's mean elements;
2. Watts' datum surface for the marginal zone.

The detailed results are presented and discussed briefly. Evidence is found for the cumulative effect of truncation in some planetary terms.

1. Corrections to Brown's values for the mean elements used in the lunar ephemeris $j = 2$; T is measured in centuries from 1900. The errors quoted are 2σ .

Mean longitude $\Delta L = -0^{\circ}.08 + 1^{\circ}.4 (T-0.63) - 10'' (T-0.63)^2$
 (referred to $\pm 0.02 \pm 0.1 \pm 3$
 'Newcomb's' equinox)

Only data after 1955.5 was used for this unknown, with
 $\Delta T = 32^s.24 + (IAT-UT1)$

The addition of $-10''T^2$ to the value $-11^{\circ}.22T^2$ in $j = 2$ gives the 'tidal' deceleration as $-21''T^2$

Longitude of perigee $\Delta \tilde{\omega} = -0^{\circ}.76 - 2.0 (T-0.63)$
 $\pm 0.04 \pm 0.4$

Longitude of node $\Delta\Omega = +1''.9 - 1'' \text{ (T-0.63)}$

$$\pm 0.1 \pm 1$$

The correction of $-1''\text{T}$ does not significantly reduce the outstanding difference of about $8''\text{T}$ between the observed and theoretical values.

Eccentricity $\Delta e = 0''.00 \pm 0.02$

Inclination $\Delta i = -0''.23 \pm 0.04$

The coefficients for eccentricity and inclination are highly correlated with that for the separation between the centres of mass and figure of the Moon in the direction towards the Earth (dependent on libration). I have assumed, in this solution, that the centre of mass is 2 km nearer the Earth than the centre of Watts' datum for the marginal zone.

Longitude of perigee of Sun $\Delta\omega' = -9'' \pm 3$

Eccentricity of Sun's orbit $\Delta e' = -0''.15 \pm 0.05$

2. Corrections to Watts' datum surface of marginal zone.

Correction in latitude to Watts' centre of figure to obtain centre of mass is $+0''.17 \pm 0.01$ (assuming $\Delta\delta_* = 0$ in FK4 star system).

Correction to reduce Watts' datum surface to a sphere is $-0''.09 (1 - 0.031^\circ) \cos 2 (\pi - 130^\circ)$, where 1° is the libration in longitude in degrees and π the position angle from the Moon's axis.

Periodicities in residuals provisionally identified with
planetary terms in latitude and longitude

| ILE No. | Coefficient | Argument | Approximate amplitude of correction |
|-----------|-------------|--------------------------|---|
| Latitude | | | |
| 1235 | 0.077 | $+L_{\odot} + 5T - 3V$ | 0.05 |
| 1240 | 0.074 | $+L_{\odot} - 5T + 3V$ | 0.05 |
| 1195 | 0.045 | $-F + 2D - 3T + 3V$ | 0.03 |
| 1224 | 0.003 | $+2I + F - 2D + 3T - 3V$ | 0.03 |
| Longitude | | | |
| 965 | 0.144 | $+I + T - J$ | 0.02 |
| (966 | 0.158 | $+I - T + J$ | not estimated because period close to D) |
| 970 | 0.062 | $+I + J$ | |
| 975 | 0.063 | $+I - J$ | present? |

SELENOGRAPHIC CONTROL

Michael Moutsoulas

Evaluation of selenographic data obtained with use of different observational means require the formulation of rigorous algorithms connecting the systems of coordinates, which the various methods have been referred to. The lunar principal axes of inertia are suggested as most appropriate for reference in lunar mapping and selenographic coordinate catalogues. The connection between the instantaneous axis of lunar rotation (involved in laser ranging, radar studies, astronomical observations from the surface of the Moon and VLBI observations of ALSEPs), the ecliptic system of coordinates (which in reductions of observations was considered as fixed in space), the "Cassini" mean selenographic coordinates (to which physical libration measures were referred), the lunar principal axes of inertia and the invariable plane of the solar system is discussed.

A NEW LUNAR EPHEMERIS INCORPORATING
LASER RANGING DATA

J. Derral Mulholland and Peter J. Shelus

Analysis of lunar laser ranging data is underway at several institutions. We describe here our efforts at improving the numerical ephemeris of Moon, based on over three years' span of data. Data identification procedures, orbit generation and correction are given in moderate detail. Comparisons of the new ephemeris with observations and with more standard ephemerides are illustrated.

ET-UT FROM 1000 BC TO THE PRESENT VERSUS THE SECULAR ACCELERATION OF THE MOON

Paul M. Muller

In this paper it is shown that there is no conclusive evidence from ancient or medieval observations to indicate that the secular (tidally-induced) acceleration of the moon has ever differed from the currently accepted value of $-11.22'' \cdot T^2$ ($-22.44''/\text{cy}^2$) used in many national ephemerides and attributed to Spencer Jones. My re-analysis of the data yields results between -20 and $-80''/\text{cy}^2$ depending on the data and which selected. These results are consistent with either a fixed or variable \dot{n}_m in the range above, and no strong solution for this parameter can be made from the available data in my opinion. Munk and MacDonald have found no theoretical mechanism to explain any significant variation in this parameter. Under the assumption that the lunar acceleration has always had Spencer Jones value, the modern, medieval, and ancient observations cited by R. R. Newton and others including the author are all combined to yield a definitive determination of ET-UT (difference of ephemeris and universal times) over the period 1000 BC to 1715 AD. The resulting observed variations in ET-UT are consistent with the contention of Munk and MacDonald (and others) that short (decade) and medium (century) term variations in the earth's rotation

dominate over periods of time shorter than millenia. This means that the current lunar ephemeris (Improved Lunar Ephemeris or LE_1) with the deduced values of $ET-UT$ is consistent with the observations and theoretical analyses thus far considered (with a few exceptions), and it is offered as the best solution for these parameters available at the present time.

REDUCTION OF EPHEMERIS ERROR INFLUENCE IN
DETERMINATIONS BY LUNAR LASER RANGING

A. Orszag

It has been shown in different previous papers that continuous or quasi continuous ranging at a lunar reflector from a terrestrial observatory will yield the distance w of that observatory to earth instantaneous axis of rotation, and its longitude L with respect to Ephemeris meridian. However it has been shown also that these determinations imply a very accurate knowledge of the geocentric range of the same reflector. By developing the difference: true minus computed geocentric ranges as a function of time, we establish relations between the first and second order terms of the above development and the errors entailing the longitude and distance to axis determinations, respectively.

On the other hand, the different terms of that same development are related to the first, second, ... variations of true minus computed geocentric ranges of the reflector during lunar passage. Thus, the first two differences are finally related to errors committed on L and w determinations.

The above properties have been extended to higher order variations. We show that, as could be expected, those with odd order correlate with longitude errors, and those with even order with distance to axis errors.

Inasmuch as the magnitude of successive terms can be expected to decrease as their order increases, it appears thus that the most accurate determinations for L and w should rely on higher order variations, calling for range measurements in large number and evenly distributed during moon passage.

At the same time, comparison between w and L values resulting from variations of the same parity but different orders will provide a coherence test for such determinations.

PHYSICAL LIBRATIONS OF THE MOON
BY NUMERICAL INTEGRATION

Haim B. Papo

The differential equations of rotational motion of the moon are solved by numerical integration. Euler's dynamical equations transformed to a convenient form are treated by techniques analogous to ordinary orbit determination procedures. The proposed method is consistent with a numerical ephemeris of the moon and can utilize a variety of observational material for the solution of the selected parameters. The parameters are grouped into the following three groups:

1. The physical libration angles of the moon and their time rates at an arbitrary epoch.
2. Physical constants featuring the principal moments of inertia of the moon.
3. Parameters associated with the particular observational material being used.

Examples are brought of comparisons between the proposed method and Eckhardt's 1970 model of the physical librations of the moon. The merits of the new method are discussed in the light of conventional data sources like earth-based or satellite-based photography as well as newly available data types like laser ranging to retroreflectors on the moon.

SOME EFFECTS OF ELASTICITY ON LUNAR ROTATION

S. J. Peale

A general Hamiltonian for a rotating moon in the field of the earth is expanded in terms of parameters orienting the spin angular momentum relative to the principal axes of the moon and relative to coordinate axes fixed in the orbit plane. The effects of elastic distortion are included as modifications of the moment of inertia tensor, where the magnitude of the distortion is parameterized by the Love number k_2 . The principal periodic terms in the longitude of a point on the moon due to variations of the tide caused by the earth are shown to have amplitudes between 3.9×10^{-3} and 1.6×10^{-2} with a period of an anomalistic month and 2.4×10^{-4} and 9.6×10^{-4} with a period of one half of a nodical month. The extremes in the amplitudes correspond to rigidities of 8×10^{11} cgs and 2×10^{11} cgs respectively, the former rigidity being comparable to that of the earth. Only the largest amplitude given above is comparable to that detectable by the projected precision of the laser ranging to the lunar retroreflectors, and this amplitude corresponds to an improbably low rigidity for the moon.

A detailed derivation of the free wobble of the lunar spin axis about the axis of maximum moment of inertia is given, where it is shown that elasticity can alter the period of the

free wobble of 75.3 years by only 3×10^{-4} to 10^{-3} of this period. Also, the effect of elasticity on the period of free libration is completely negligible by many orders of magnitude.

If the moon's rigidity is close to that of the earth there is no effect of elasticity on the rotation which can be measured with the laser ranging, and therefore no elastic properties of the moon can be determined from variations in the rotation.

ON THE SHAPE OF THE MOON AND
ITS PHYSICAL IMPLICATIONS

S. K. Runcorn

The method of geometrical librations if statistically treated provides information about the global shape of the lunar nearside. The general conclusions are in agreement with the much more accurate determinations of the heights of the limited parts of the lunar surface obtained by space technology especially the laser ranging in the Apollo 15 and 16 missions. Important conclusions can be derived concerning the physical processes in the interior of the Moon from the comparison of the global shape of the Moon and the second harmonic of its gravitational field. Explaining the origin of the mascons by the upwelling of lava into the large impact-produced basins on the nearside provided a means of explaining the one great discrepancy between the shape of the Moon determined by ground based astronomy and space techniques, i.e. the difference between the centre of figure of the Moon and its centre of mass. This phenomena, however, must be distinguished from the discrepancy between the two sets of measurement in which the lengths of the lunar radii obtained from the geochemical libration data are about 2 kms. greater than those determined by the space techniques. This seems to arise from an incorrect mean radius of the Moon.

COORDINATE SYSTEMS AND LUNAR OBSERVING STATION POSITIONS

Joseph W. Siry

Satellite geodesy has yielded the locations of more than fifty stations in a single coordinate system referred to the earth's center of mass with accuracies in the five to ten meter range. The following different methods have been used at Goddard to accomplish this.

Dynamical solutions have been obtained for the locations of some fifty key stations using data from the GEOS satellite program. The distribution of observations about the stations is illustrated in terms of the data obtained for a typical station such as the one at Edinburg, Texas. Geopotential coefficients were held fixed in these solutions. The results of these dynamical determinations implied geodetic datum shifts which were then used to arrive at positions for some two hundred additional stations.

Another approach involved the adjustment of the coordinates of seventeen stations on the basis of observations of short arcs of GEOS satellite orbits. These results were found to be consistent with those obtained through ground surveys to about five meters rms in each coordinate.

Simultaneous solutions for station locations and geopotential coefficients have also yielded values for positions of some sixty stations, again in a coordinate system defined in terms of the earth's center of mass.

Lunar laser ranging and lunar occultation observing programs involve knowledge of the positions of the observing sites. In some cases the lunar observing program itself yields station coordinate information. In other cases greater reliance is placed upon independent determinations of site locations. The location of an occultation observation site at Olifantsfontein, for example, has been obtained in a center-of-mass system in both the dynamical and simultaneous satellite solutions. It is anticipated that a dynamical satellite solution will be extended in 1973 to obtain center-of-mass coordinates for a station in New Zealand. This will make it possible to tie an occultation site in that region to a dynamically determined coordinate system referred to the mass center. Coordinates for stations at Organ Pass, New Mexico, determined in both the dynamical and simultaneous solutions, and Edinburg, Texas, found in both the dynamical and short-arc adjustments, provide the basis for referring the location of a facility such as the McDonald Observatory to a center-of-mass system either through accurate ground surveying

techniques or by means of a satellite geodesy tie. The latter approach has already been used, for example, to fix the position of an isolated site on Madagascar relative to a reference point in Africa and, in turn, to a center-of-mass coordinate system.

Estimates of the accuracies of the satellite determinations are discussed.

Theoretical aspects of coordinate systems associated with the earth and the moon are also considered.

LUNAR PHYSICAL LIBRATIONS AND LASER RANGING

J. G. Williams, P. L. Bender, D. H. Eckhardt,
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Lunar laser ranging data has been analyzed to give corrections to the lunar ephemeris, the observatory and reflector coordinates, and the libration parameters β and γ . In an effort to improve upon existing analytical libration theories, a program has been developed which numerically integrates the physical librations. Comparisons are made between analytical and numerical results with the moon's gravitational field truncated at the second and fourth degree harmonics. The effects on librations of the higher degree gravitational harmonics, the additive and planetary terms in the lunar theory, and limitations to the determination of the lunar damping constant are discussed.

FIGURE AND ALTITUDE PROFILES FROM LASER ALTIMETRY

W. R. Wollenhaupt

The laser altimeter measurements obtained during the Apollo 15 mission provided the first set of accurate elevation differences around the entire circumference of the moon. Two additional sets, in different orbital planes, were obtained during the Apollo 16 and 17 missions. These data reveal many interesting details and indicate that the figure of the moon is very complex. For example, the lunar near side appears to be relatively smooth, having radius values much lower than the usually accepted value of 1738 kilometers. The only exception to this is the region around Descartes, which has a radius of about 1738 kilometers. The near-side ringed maria are relatively flat and are depressed with respect to surrounding terrain. Further, the depth of these maria apparently increases from west to east, the deepest being Mare Smythii. The far side is extremely rough in comparison to the near side, much rougher, in fact, than was expected prior to obtaining the Apollo 15 measurements. A large depression on the far side was observed in both the Apollo 15 and 17 data. This was also unexpected.

The best least-squares fit to the laser data indicates that the moon is essentially a sphere of radius 1738 kilometers.

In addition, the laser data prove the existence of a displacement between the lunar center of gravity and center of figure. The current best estimate is that the center of gravity is displaced about 2 kilometers toward the earth and 1 kilometer eastward from the center of figure. The figure parameters in the polar direction are not well determined.